[redacted]

SEMI-ANNUAL TECHNICAL REPORT

to the

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

from

Eugene Herrin

Geophysical Laboratory
Institute for the Study
of Earth and Man
Southern Methodist University

For the period ending September 1, 1975

ARPA Order: 2382
Program Code: 4F10
Name of Contractor: Southern Methodist University
Effective Date of Contract: January 16, 1974
Contract Expiration Date: July 15, 1976
Amount of Contract Dollars: $711,731
Contract Number: F 44620-73-C-0044
Principal Investigator and Phone Number: Eugene Herrin, #214-692-2760
Program Manager and Phone Number: Truman Cook, Director of Research Administration, #214-692-2031
Title of Work: Improved Methods for Detection of Long Period Rayleigh Waves and for Identification of Earthquakes and Underground Explosions

Sponsored by
Advanced Research Projects Agency
ARPA Order No. 2382

REMOVAL OF MULTIPATH EFFECTS
IN THE ESTIMATION OF RAYLEIGH WAVE
SPECTRA

Eugene Herrin
Tom Goforth

ABSTRACT

The effects of multipathing on the Rayleigh spectrum mask depth-dependent spectral zeroes and cause significant errors in magnitude determinations. A new technique, called phase equilization filtering, is introduced for the purpose of eliminating multipath effects. The technique is demonstrated to be effective on synthetic signals in which the undisturbed spectrum is known. In a typical case, an 18-dB spectral hole at a period of 20 seconds, caused by multipathed energy arriving only 30 seconds later than the primary arrival, was essentially eliminated. The usefulness of the technique is also demonstrated by application to teleseismic Rayleigh waves.

Introduction

The presence of non-horizontal boundaries in the crust and upper mantle of the earth result in reflection and refraction of Rayleigh and Love waves. Secondary arrivals, or multipaths, are produced, and these cause constructive interference at some frequencies in the recorded signal and destructive interference at other frequencies. The effect of multipathing on the surface wave is sufficiently great to mask or distort signal characteristics which are important in the analysis of seismic events, especially those originating in the structurally complicated Sino-Soviet sub-continent. Multipathing and related effects of the propagation path, unless identified and removed, will limit the success of research efforts directed toward identifying source differences between explosions and earthquakes, in utilizing the improved horizontal data expected from borehole long period instruments, and in achieving accurate yield determinations from recordings of surface waves.

In this report we discuss a new approach to the removal of the effects of multipathing from surface wave signals. We propose to develop and to apply this technique, called phase equilization filtering, to Rayleigh and Love waves originating from events in the Asian continent.

Phase Equalization Filtering

If the phase of a dispersed wave train is known or can be determined, a filter can be developed which has the same phase spectrum as the dispersed signal and any desired amplitude spectrum. Cross-correlation of the signal and the phase equalized filter produces a function, called a pseudo-autocorrelation function (PAF), which possesses several desirable properties. The PAF contains the spectrum of the signal in a retrievable form. Time compression of the signal produces a S/N improvement on the order of 10 dB and allows better separation of the primary and multipaths or second signals. Mathematically, the PAF is an even function and has many of the desirable properties of the auto-correlation function.

Let $c(t)$ be the filter obtained from an estimated dispersion curve and $g(t)$ the recorded signal. Then

$$c(t) \star g(t) = \int_{-\infty}^{+\infty} g(t) \cdot c(t+T)\,dt = H(T)$$

where $H(T)$ is the cross-correlation function. The Fourier transform of $H(T)$ can be written

$$G(\omega) \cdot C(\omega) \ \exp i\left[\theta_g(\omega) - \theta_c(\omega)\right]$$

or

$$G(\omega) \cdot C(\omega) \left\{ \cos\left[\theta_g(\omega) - \theta_c(\omega)\right] + i \sin\left[\theta_g(\omega) - \theta_c(\omega)\right] \right\}$$

Clearly the real part of the transform of $H(\tau)$ will be maximized when the phase spectra of the filter and the signal are equal. In this case the spectrum $G(\omega)C(\omega)\cos(0)$ will be real and even, and $H(\tau)$ is defined to be a PAF. The process of maximizing the integral of the real spectrum corresponds in the time domain to maximizing the peak value of the correlation function. If no multipathing is present, this maximization can be achieved in practice by varying the phase of the filter until it matches that of the signal. If $g(t)$ consists of a primary and secondary signals arriving at different times, $H(\tau)$ will consist of a series of peaks corresponding to each component of the total signal, the first peak corresponding to the primary arrival. Theoretically, $H(\tau)$ endures from $-\infty$ to $+\infty$. In practice, for band-limited data (periods of 15-70 seconds, for example), the useful information is almost entirely contained in a window $\pm$ 150 seconds from the peak of the correlation function. A 300-second separation in the arrival times of a primary and a multipath or between different signals would result in noninterfering peaks and would allow the determination of the phase equalized filter by maximizing the amplitude of the primary peak. Simultaneously, the spectrum of the primary signal, undistorted by multipathing, can be obtained by transforming a 300-second window centered at the primary peak. Indeed, this process can be accomplished not only for the primary arrival, but for any multipath or

interfering secondary component making up the total signal.

As time separation between the primary and secondary arrivals decreases to less than 300 seconds, use can be made of the property of PAFs that in the absence of multipathing they are even functions. Thus, in the presence of multipathing, we know that the right-hand side of the primary peak which is disturbed by multipathing would, if the multipathing were not present, be the mirror image of the left-hand side which is not disturbed by multipathing. Replacement of the right-hand side by the reflected left-hand side of $H(\tau)$ removes the effect of the multipath, unless the time separation between the arrivals is so small that the multipath peak affects the left-hand side of the primary peak. In practice, a time separation of about 50 seconds appears to be resolvable for signals containing periods up to 70 seconds.

An earthquake occuring in Tadzhik, USSR, and recorded at the Large Aperture Seismic Array (LASA) in Montana is useful in demonstrating both the profound effects of multipathing and the effectiveness of phase equalization filtering in removing the multipaths. Figure 1 shows the vertical long-period seismograms recorded at sites A0, F1 and F3, respectively, at LASA. The average properties of the propagation paths from an earthquake 10,000 kilometers away to each of these sites, which are separated by about 100 kilometers, cannot be very different;

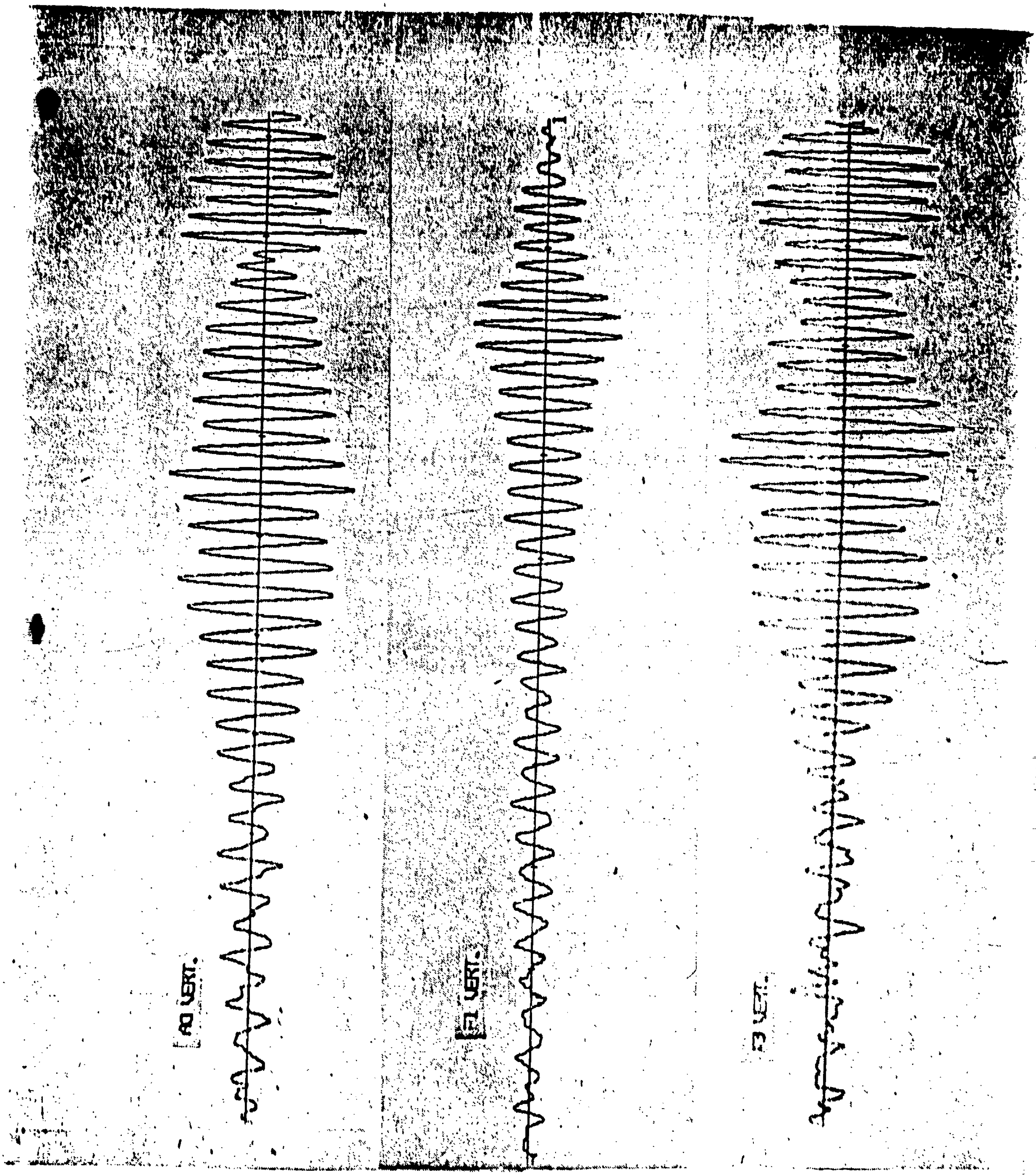


Figure 1

however, the seismograms are obviously different. The amplitude spectra of the seismograms recorded at A0, F1, and F3, shown in Figure 2, are also different. Using the criteria developed above, it was possible to find a single phase equalized filter which fit each seismogram. Correlation of this filter with each of the three seismograms, replacing the right-hand sides of the primary peaks of the resulting correlation functions with the reflected left-hand sides to remove the multipaths, taking the Fourier transform of the PAFs, and dividing the resulting spectra by the spectrum of the filter resulted in the spectra shown in Figure 3. Note that these spectra with the multipath effects removed are now very similar, as might be expected for seismograms representing almost the same propagation path. A very slight difference in propagation path can, and often does, result in dramatic differences in multipath effects.

Another example from LASA demonstrates the S/N and time compression aspects of phase equilization filtering. A small magnitude (m_b=4.7) earthquake whose epicenter was located approximately 1400 kilometers northeast of the Tadzhik event is barely recognizable on the A0 vertical LP channel, shown in Figure 4. Figure 4 also shows the function obtained by correlating the signal with the same filter determined for the Tadzhik event. The increase in S/N is apparent. Both the least time arrival and a multipath or aftershock arriving 140 seconds later are now clearly visible.

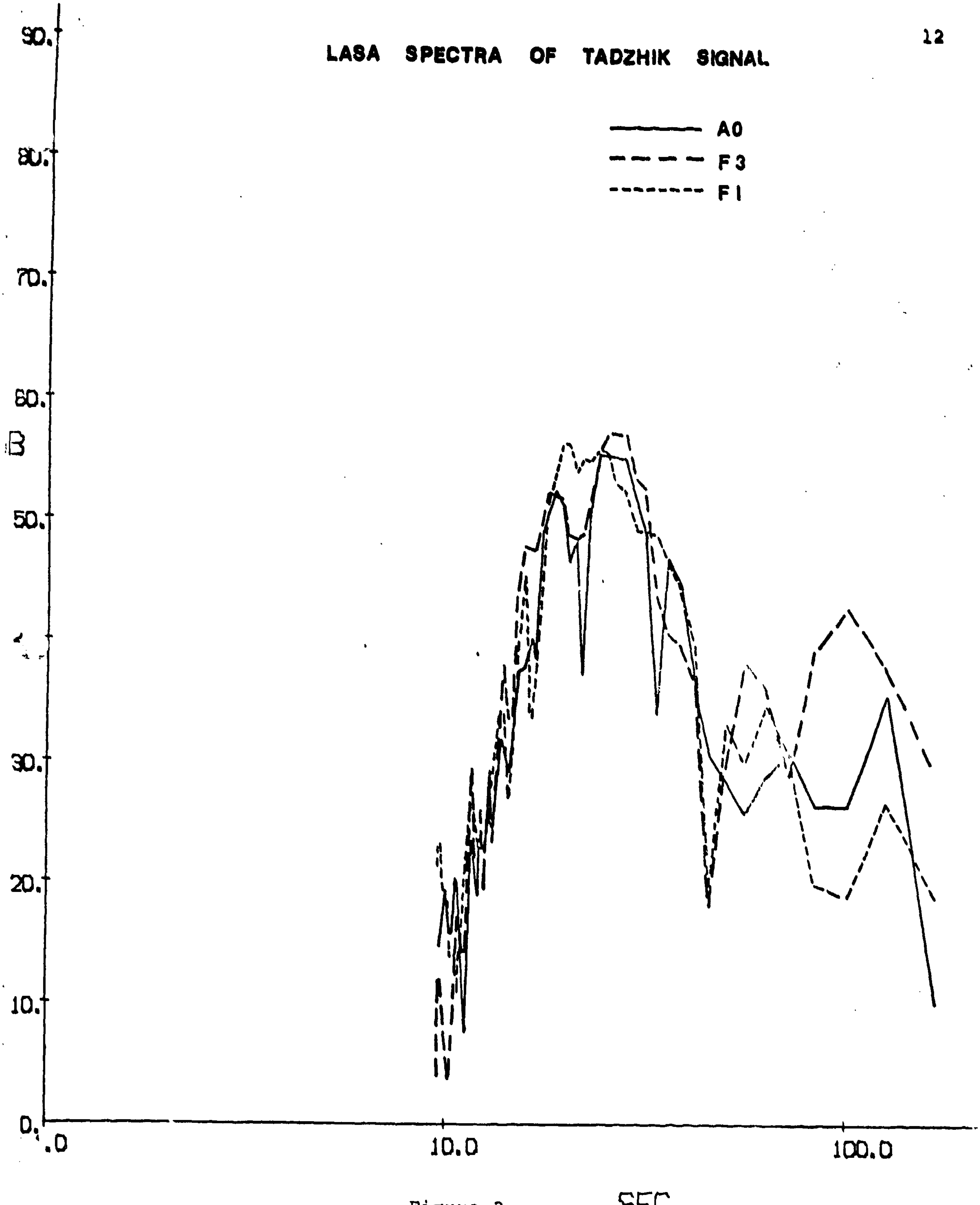

LASA SPECTRA OF TADZHIK SIGNAL
A0
F3
F1
90.
80.
70.
60.
B
50.
30.
20.
10.
0.
.0
10.0
100.0
SEC

Figure 2

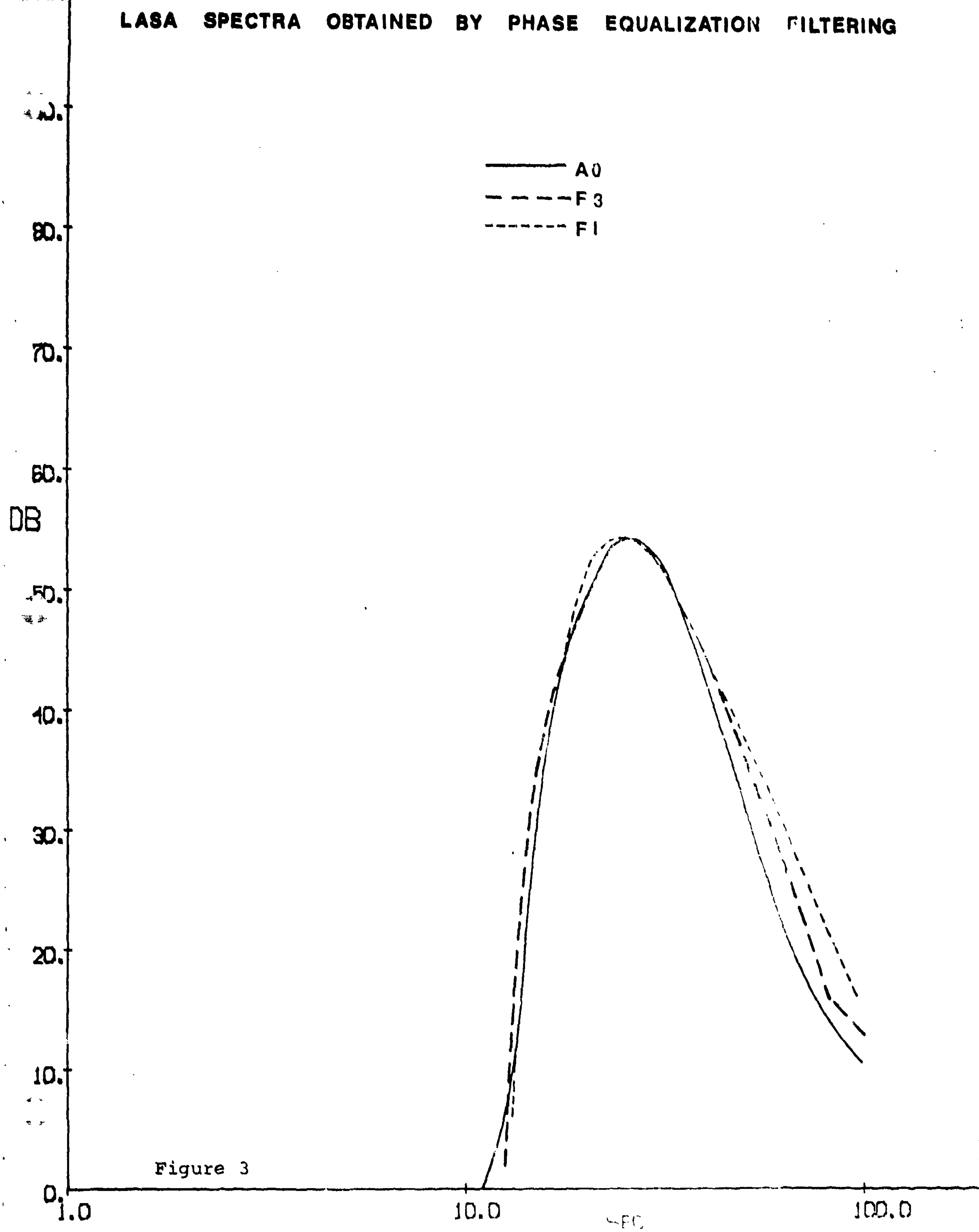

LASA SPECTRA OBTAINED BY PHASE EQUALIZATION FILTERING
A0
F3
F1
DB
100.
80.
70.
60.
50.
40.
30.
20.
10.
0.
1.0
10.0
100.0

Figure 3

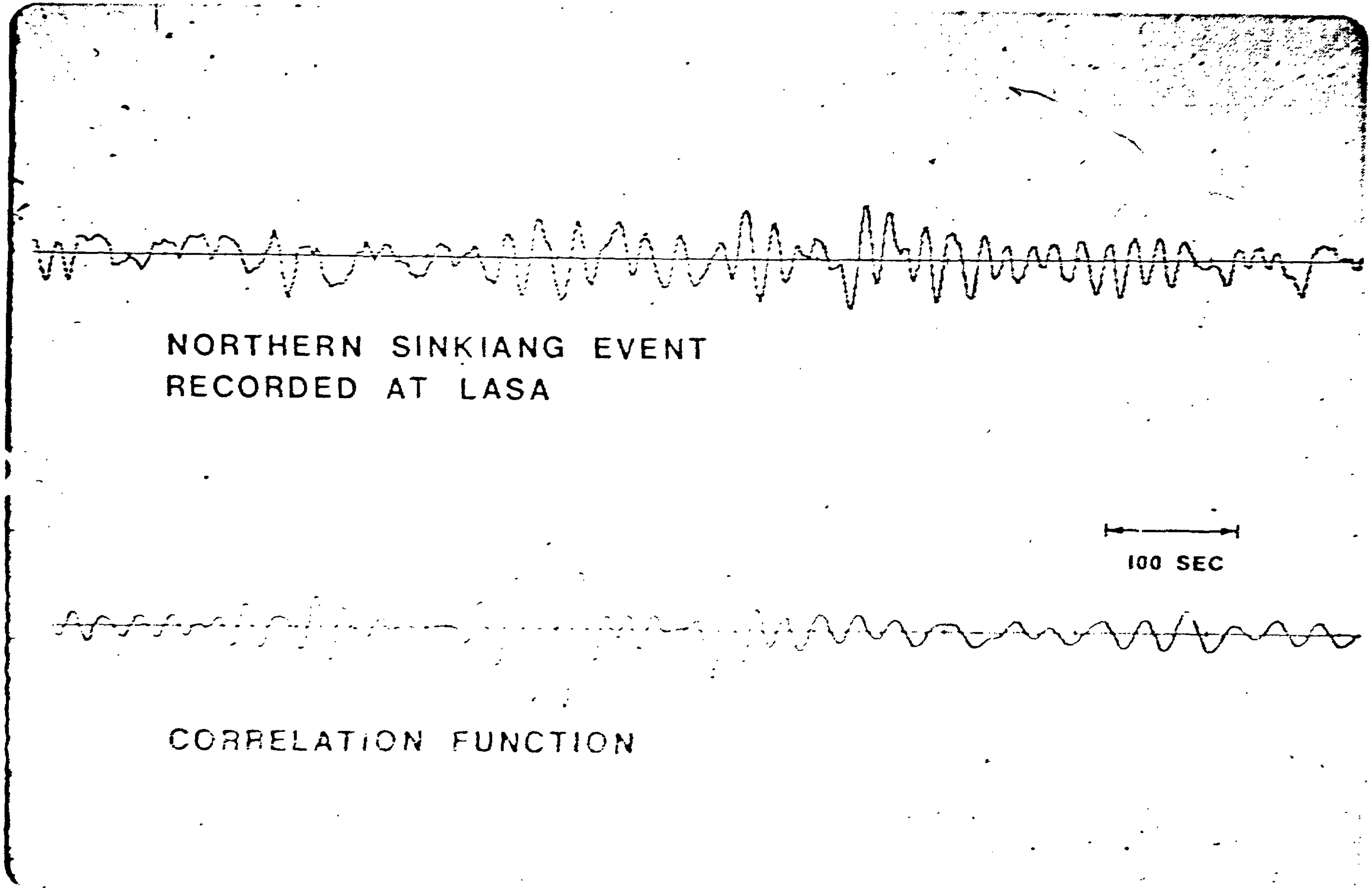

NORTHERN SINKIANG EVENT
RECORDED AT LASA
100 SEC
CORRELATION FUNCTION

Figure 4

Phase equalization filtering can be used to separate multiple events as well as primary and multipath components. A simulated mixed signal was obtained by adding the Tadzhik signal as recorded at site A0 at LASA to itself delayed by 700 seconds and reduced in amplitude by 12 dB. Figure 5 shows the real signal, the delayed and diminished version of the same signal, and the sum of the two. Note that the presence of the delayed event in the coda of the signals is not at all obvious. Using the same filter that was previously obtained for the Tadzhik signal, correlation functions were computed for both the actual and the combined signal. These are shown in Figure 6. The primary signal and a multipath with a delay of 500 seconds are obvious in the real signal correlation function, and the peak corresponding to the simulated signal at a delay of 700 seconds is also obvious in the combined signal correlation function. Time separation of the components of the combined signal is so well accomplished that it is now possible, by centering 300-second windows on the respective correlation peaks, to obtain the amplitude spectrum of the primary signal undistorted by either the multipath or the second arrival, and also the amplitude spectrum of the simulated second arrival undistorted by either the multipath or the main signal coda. These two spectra are shown in Figure 7. The spectrum of the second arrival is properly 12 dB less than that of the main signal.

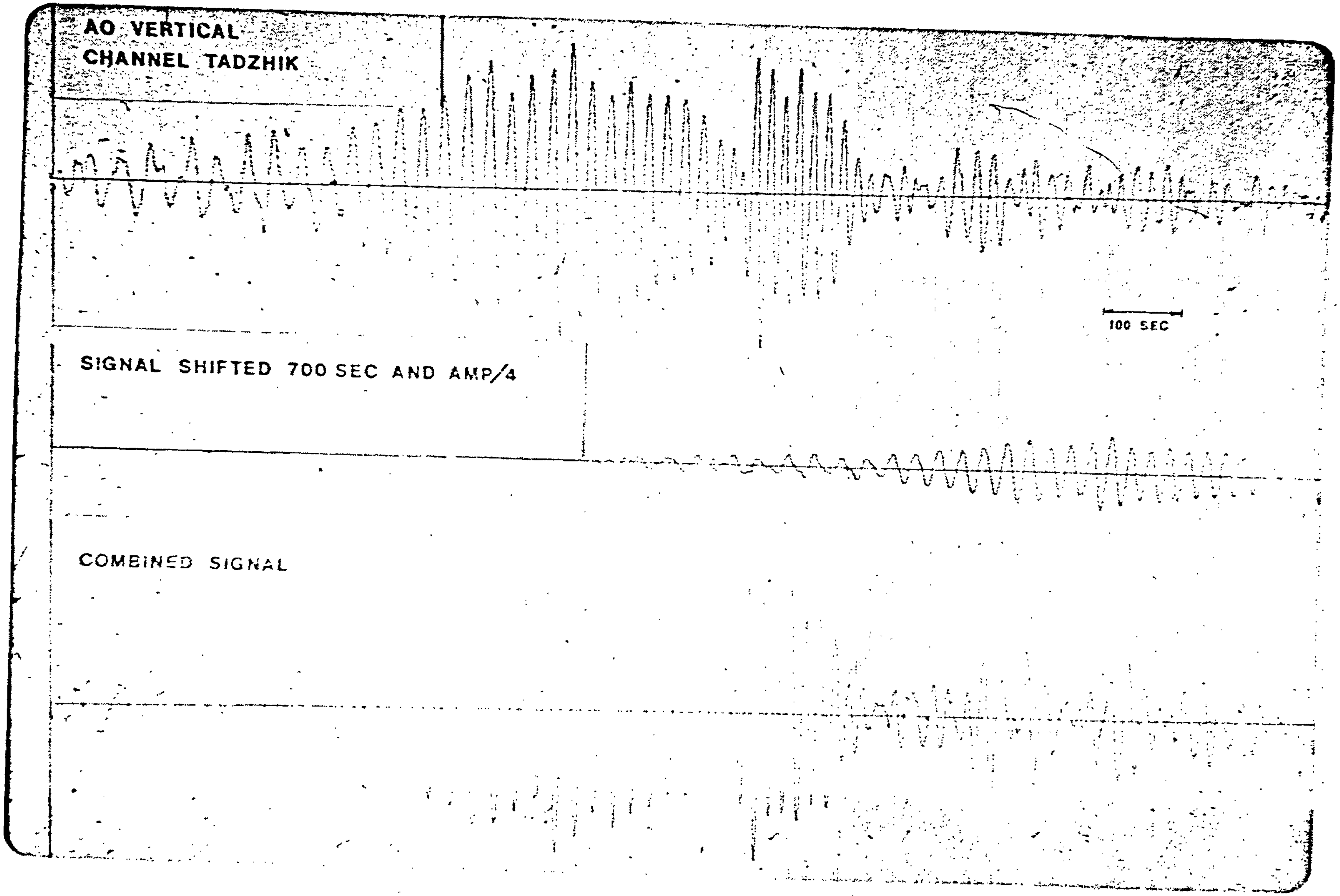

AO VERTICAL
CHANNEL TADZHIK
100 SEC
SIGNAL SHIFTED 700 SEC AND AMP/4
COMBINED SIGNAL

Figure 5

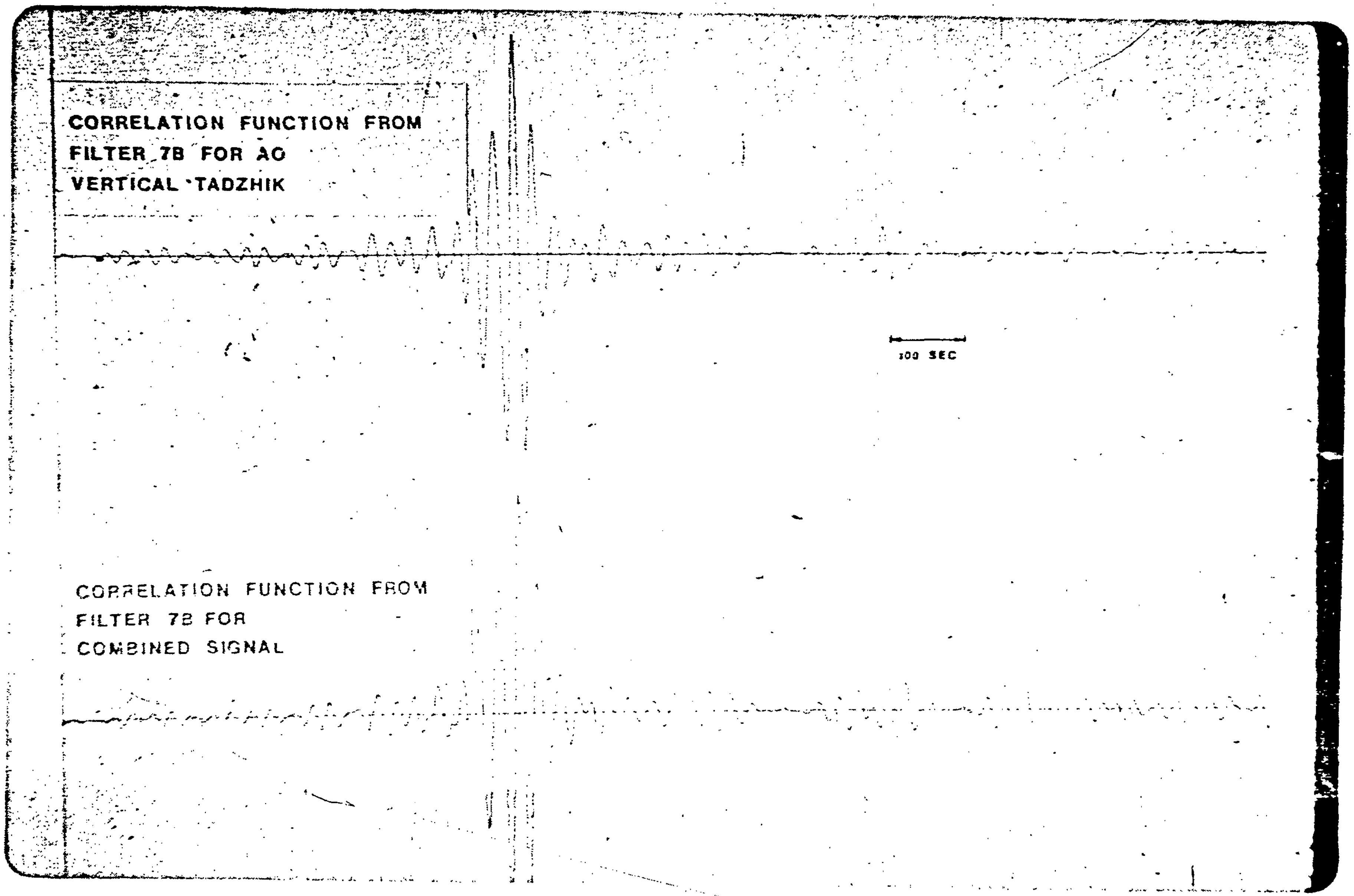

CORRELATION FUNCTION FROM
FILTER 7B FOR AO
VERTICAL TADZHIK
100 SEC
CORRELATION FUNCTION FROM
FILTER 7B FOR
COMBINED SIGNAL

Figure 6

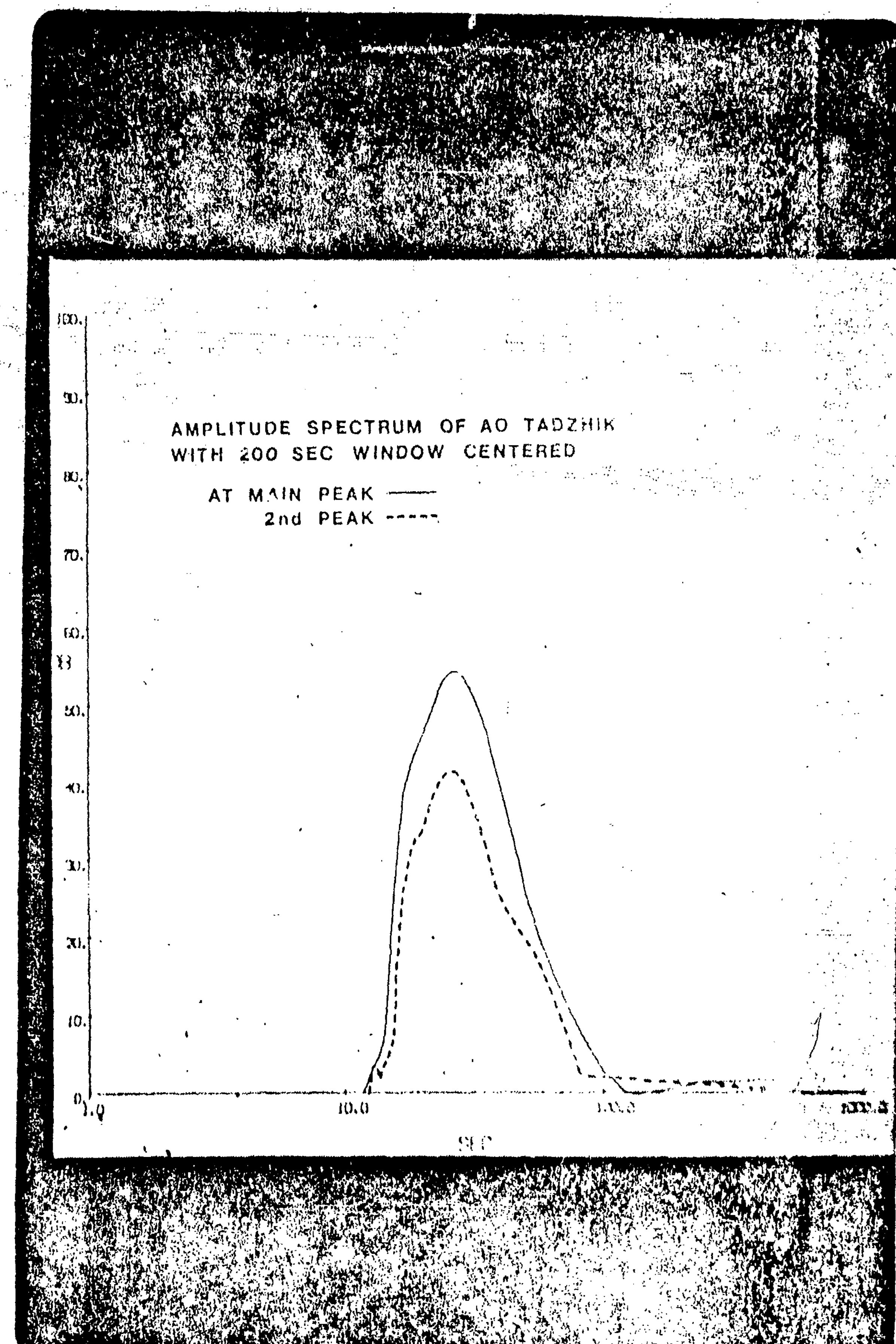

AMPLITUDE SPECTRUM OF AO TADZHIK
WITH 200 SEC WINDOW CENTERED
AT MAIN PEAK
2nd PEAK
1.0
10.0

Figure 7

Even the spectrum of the multipath, shown in Figure 8, is easily obtainable because of the time compression. The isolation and analysis of multipath spectra may make possible 'horizontal exploration seismology', the study of the location and properties of lateral boundaries in the crust and upper mantle.

It is constructive to consider the use of phase equilization filtering on synthetic signals. In this case the signal spectrum prior to multipath distortion is exactly known and can be directly compared to the filtered estimate. Consider the synthetic signal shown in the top of Figure 9. If the amplitude of the first 600 seconds of this signal is zeroed and the remainder is delayed 30 seconds and added to the original the 'multipathed' signal shown in the bottom of Figure 9 is produced. This record simulates a situation in which only signal periods less than about 30 seconds have been multipathed, and these have travelled less than 1% farther than the primary arrivals. The effect of the multipath is seen to double the amplitudes of periods around 30 seconds and almost to cancel periods around 20 seconds. Figure 10 shows the amplitude spectra of the two seismograms. A 6 dB 'build-up' at 30 seconds, a 10 dB 'build-up' at 15 seconds, and a 16 dB 'hole' at 20 seconds are obvious. A magnitude determination on this signal would be misleading because of the multipathing.

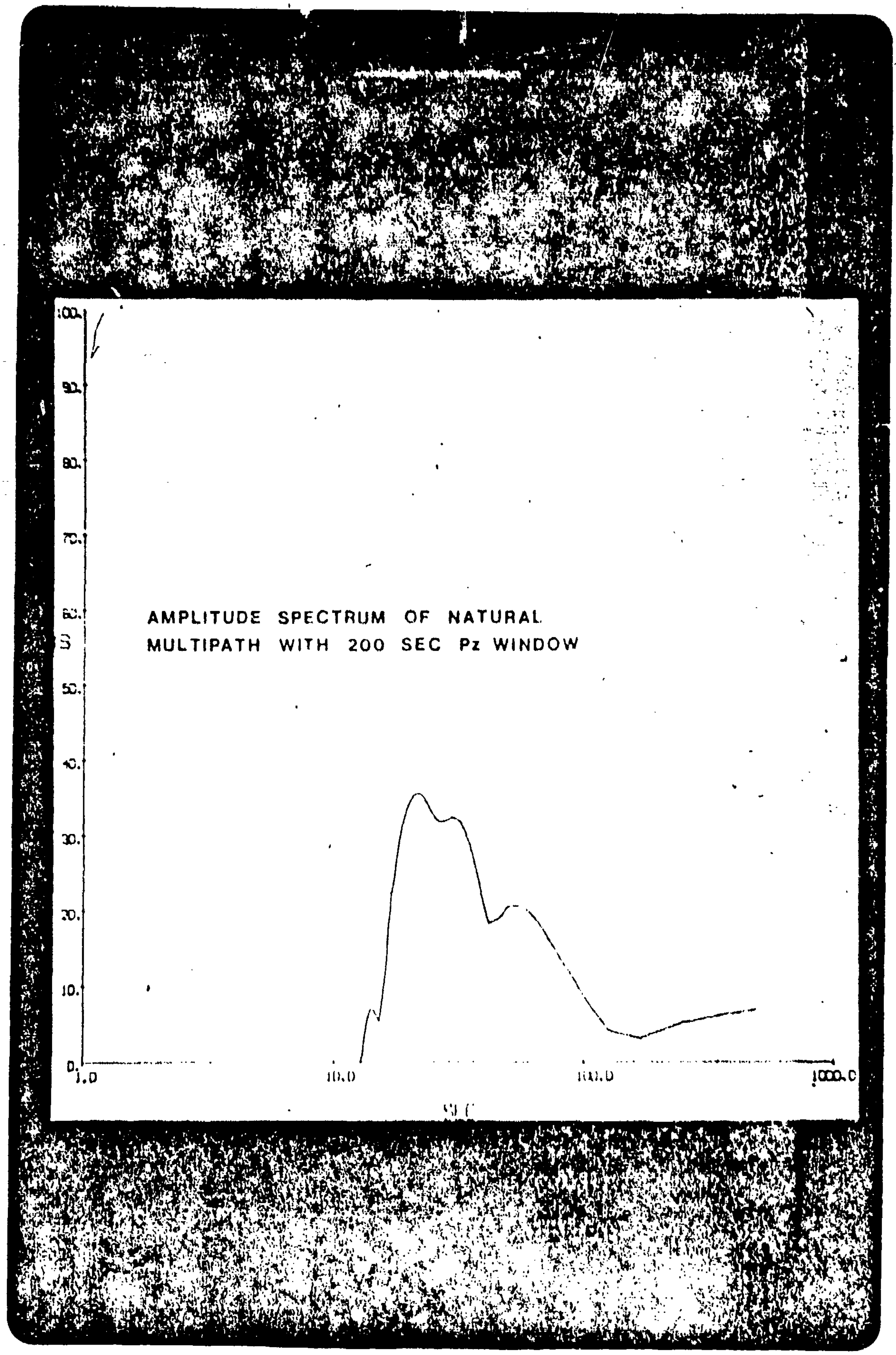

AMPLITUDE SPECTRUM OF NATURAL
MULTIPATH WITH 200 SEC Pz WINDOW
100.
90.
80.
70.
60.
50.
40.
30.
20.
10.
0.
1.0
10.0
100.0
1000.0

Figure 8

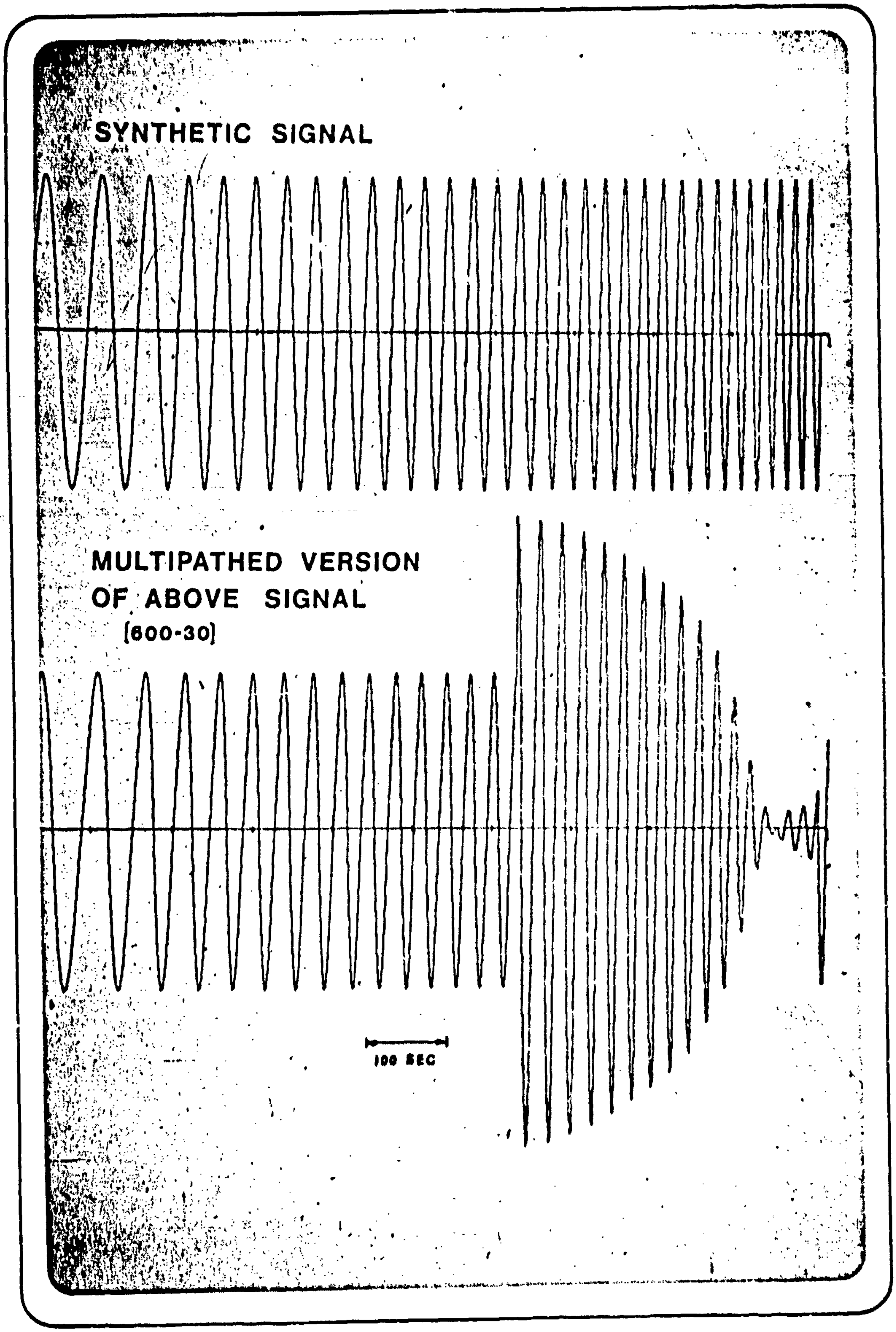

SYNTHETIC SIGNAL
MULTIPATHED VERSION
OF ABOVE SIGNAL
(600-30)
100 SEC

Figure 9

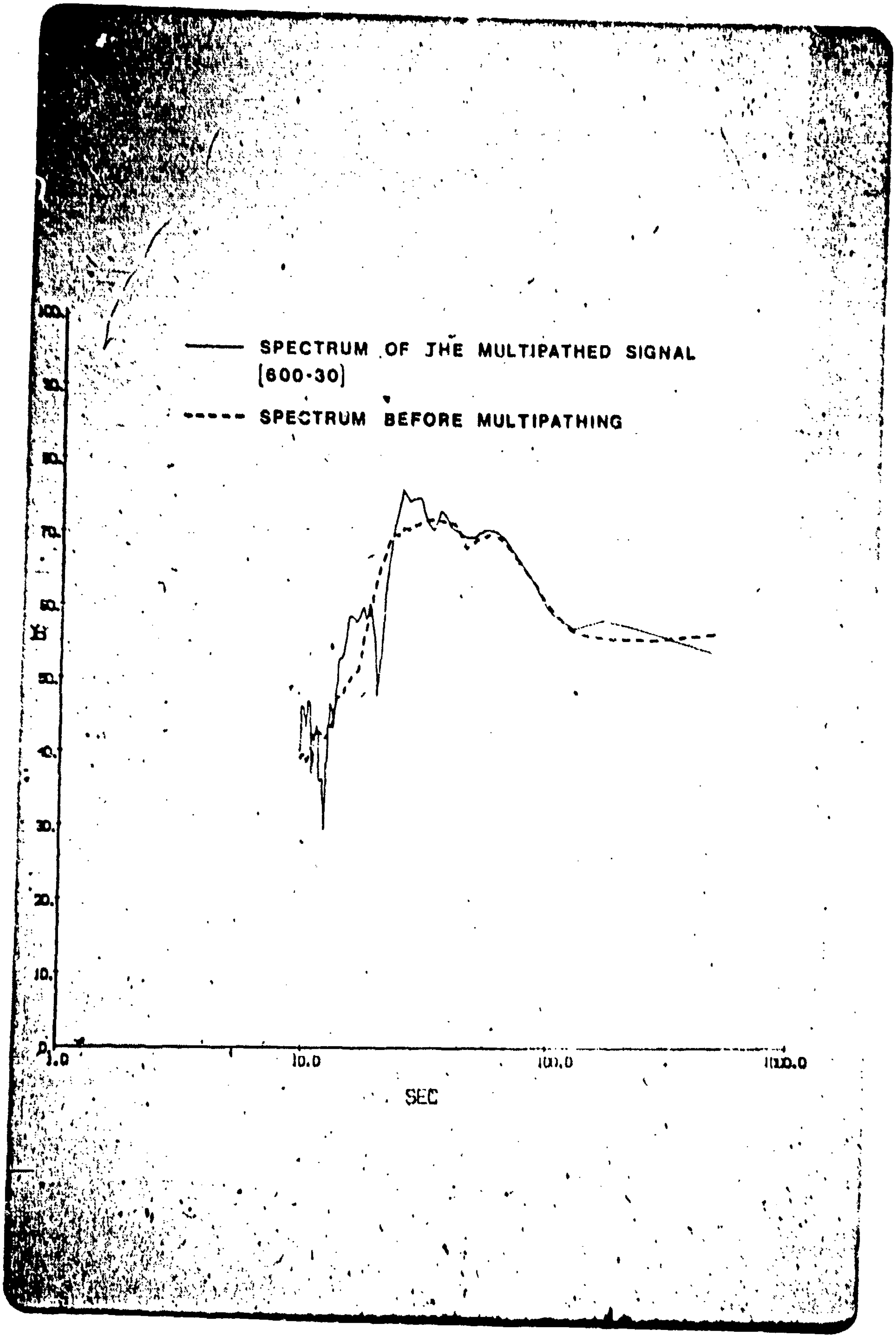

SPECTRUM OF THE MULTIPATHED SIGNAL
(600-30)
SPECTRUM BEFORE MULTIPATHING
DB
0.
10.
20.
30.
40.
50.
60.
70.
80.
90.
100.
1.0
10.0
100.0
1000.0
SEC

Figure 10

Because of the small delay (30 seconds) relative to the primary with which the multipath arrives, it is beyond the resolution of standard techniques to separate the two. For example, a multiple filter analysis of the simulated multipathed signal is shown in Figure 11. Not only are the multipath-induced build-ups and hole present, there is no hint that they are caused by a multipath.

Figure 12 shows the function obtained by correlation of the multipathed signal and the phase equalized filter; the filter for the simulated signal of course is known exactly. Also shown is the PAF, i.e., the correlation function with the right-hand side replaced by the reflected left-hand side. The spectrum of the PAF normalized for the filter spectrum is shown in Figure 13, with the spectrum of the multipathed signal also presented for comparison. The build-ups at 15 and 30 seconds and the hole at 20 seconds have been strongly attenuated.

To demonstrate that phase equalization filtering will retain spectral holes which might be present prior to multipathing, it is useful to consider another simulated situation. Figure 14 shows a synthetic signal with a time-domain modulation which will produce a spectral hole. The hole can be thought of as being produced by the source. If the amplitude of the first 100 seconds of this signal is zeroed and the remainder is delayed 70 seconds and added to the original, the 'multipathed'

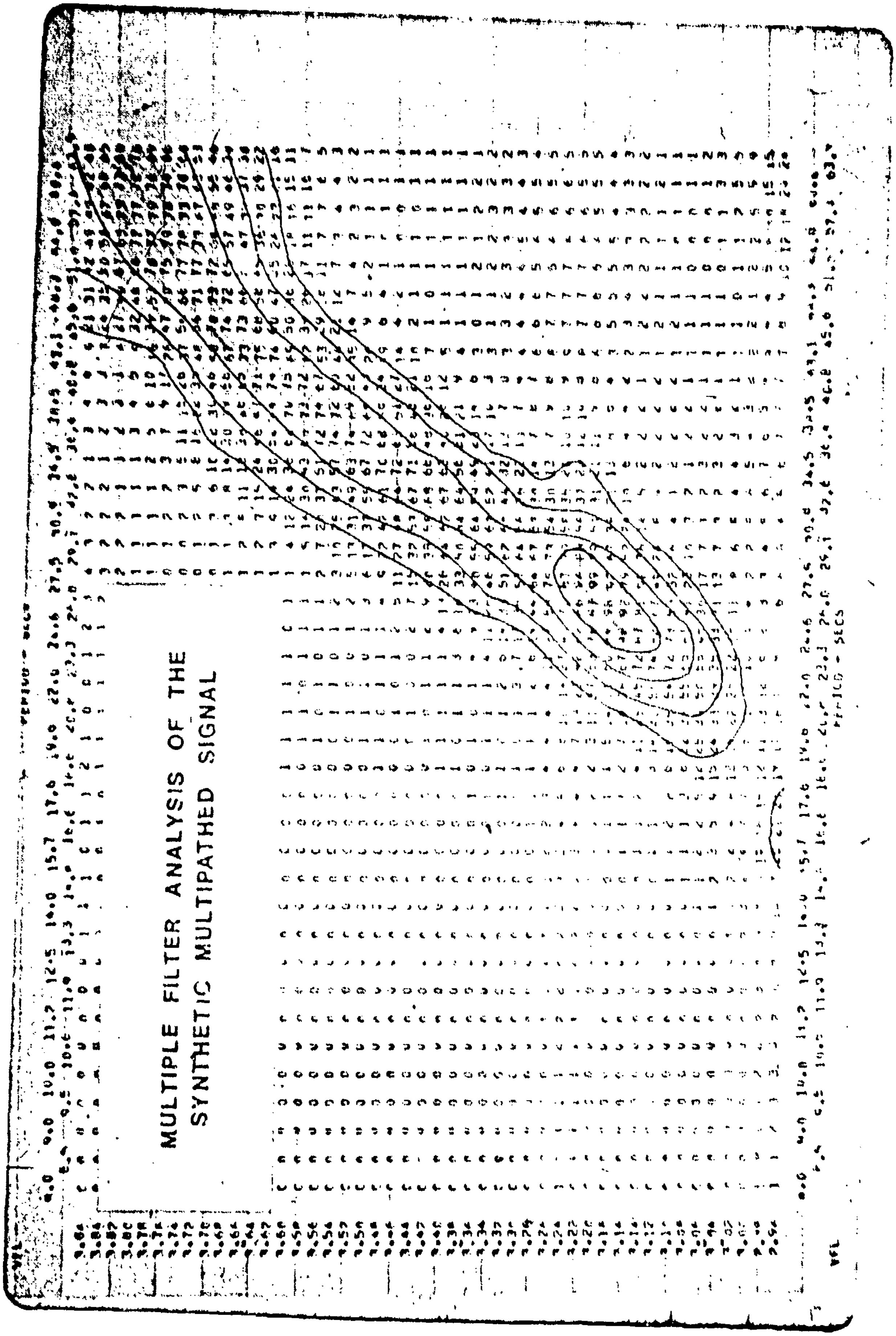

MULTIPLE FILTER ANALYSIS OF THE
SYNTHETIC MULTIPATHED SIGNAL
PERIOD - SECS

Figure 11

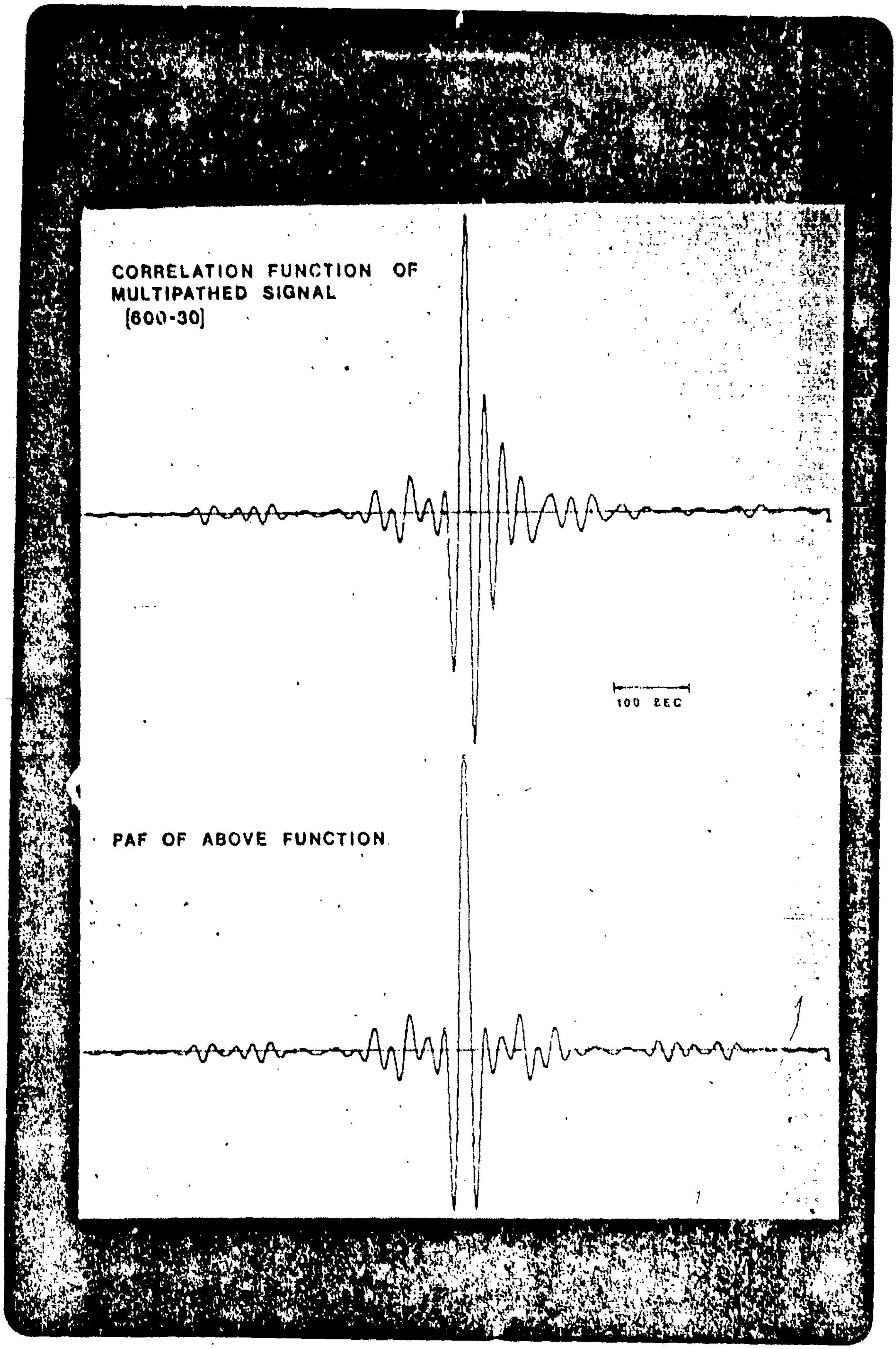

CORRELATION FUNCTION OF
MULTIPATHED SIGNAL
[600-30]
100 SEC
PAF OF ABOVE FUNCTION

Figure 12

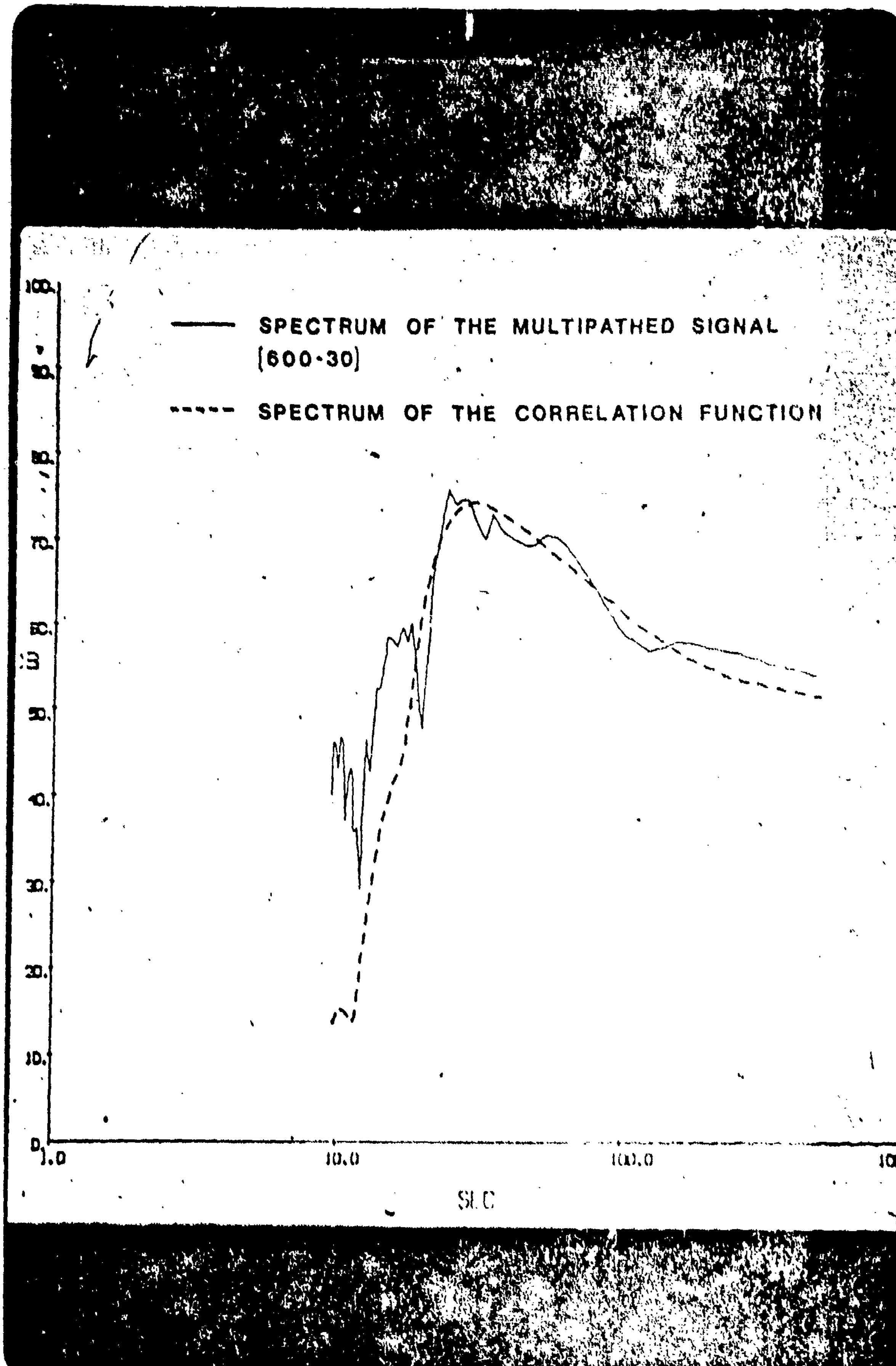

SPECTRUM OF THE MULTIPATHED SIGNAL [600-30]
SPECTRUM OF THE CORRELATION FUNCTION
0.
10.
20.
30.
40.
50.
60.
70.
80.
90.
100.
1.0
10.0
100.0
SEC

Figure 13

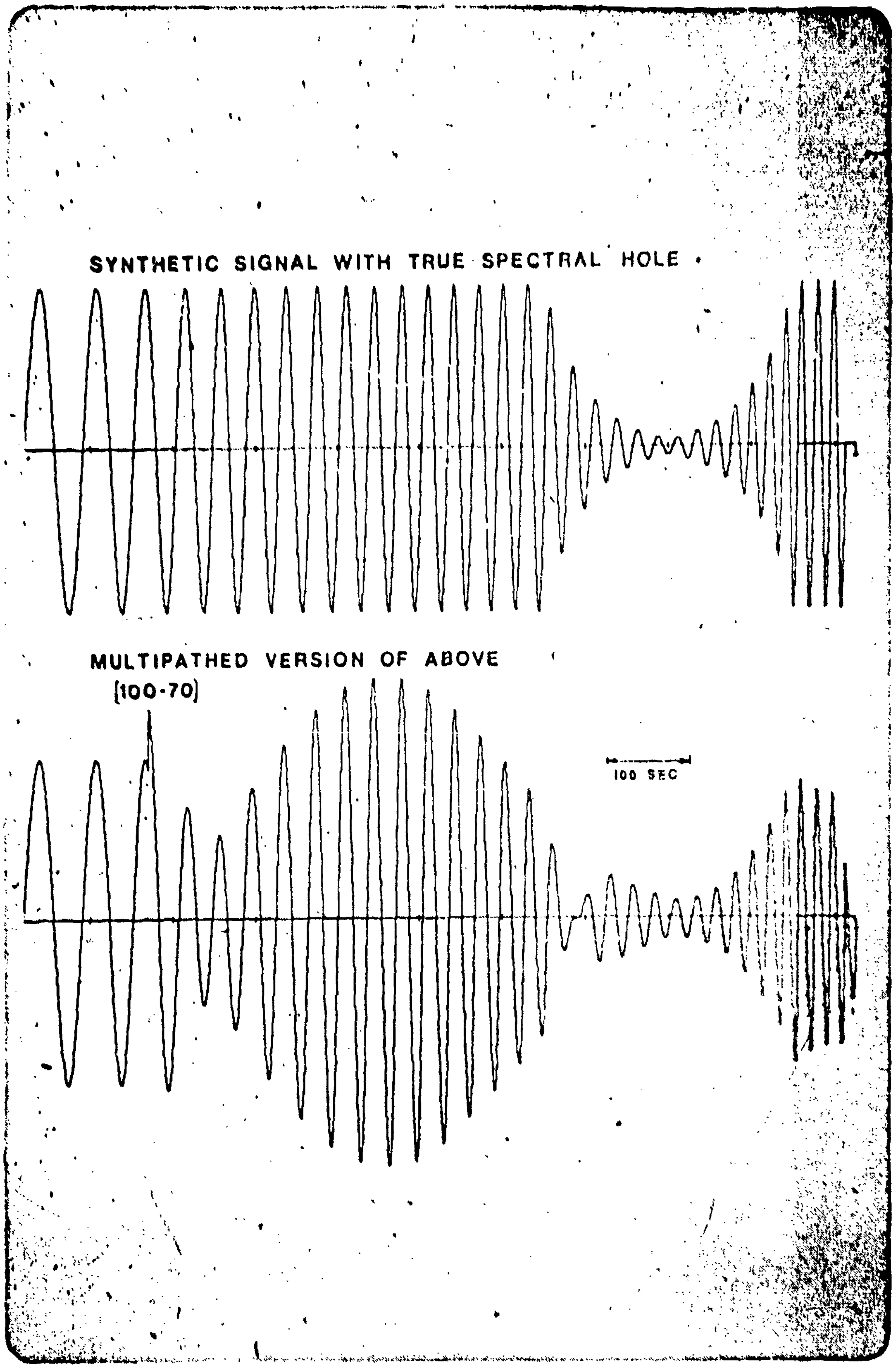
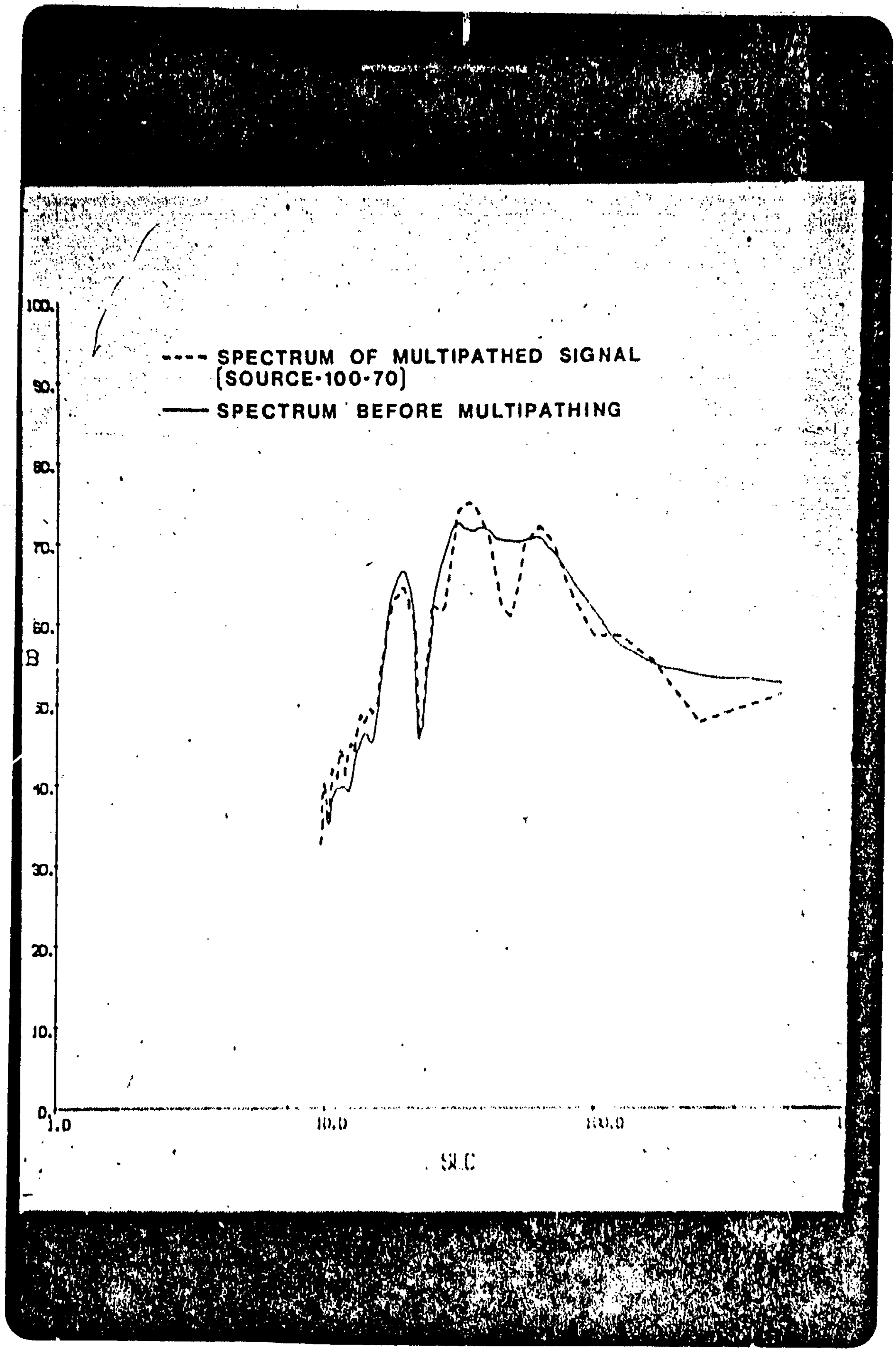

SYNTHETIC SIGNAL WITH TRUE SPECTRAL HOLE
MULTIPATHED VERSION OF ABOVE
[100-70]
100 SEC

Figure 14

signal shown in the bottom of Figure 14 is produced. The spectra of the two seismograms are shown in Figure 15. The source hole at a period of 22 seconds is relatively unaffected by the multipath, but another significant hole is introduced at a period of about 50 seconds. A correlation function is obtained by cross-correlating the multipathed signal and the phase equalized filter. This function and the corresponding PAF are presented in Figure 16. The spectrum of the PAF, along with the spectrum of the multipathed signal, is shown in Figure 17. Nearly all of the multipath hole has been eliminated from the PAF spectrum; nearly all of the source hole has been retained, and the period at which the source hole occurs has been distorted by less than a second.

These preliminary experiments using both real and synthetic data indicate that phase equalization filtering has great potential in removing effects which are undoubtedly causing scatter and error in Rayleigh wave discrimination criteria and which are masking spectral manifestations of the source. The technique is completely applicable to Love waves and can be applied as high-quality, horizontal data becomes available.



---- SPECTRUM OF MULTIPATHED SIGNAL
(SOURCE-100-70)
—— SPECTRUM BEFORE MULTIPATHING
100.
90.
80.
70.
60.
50.
40.
30.
20.
10.
0
1.0
10.0
100.0

Figure 15

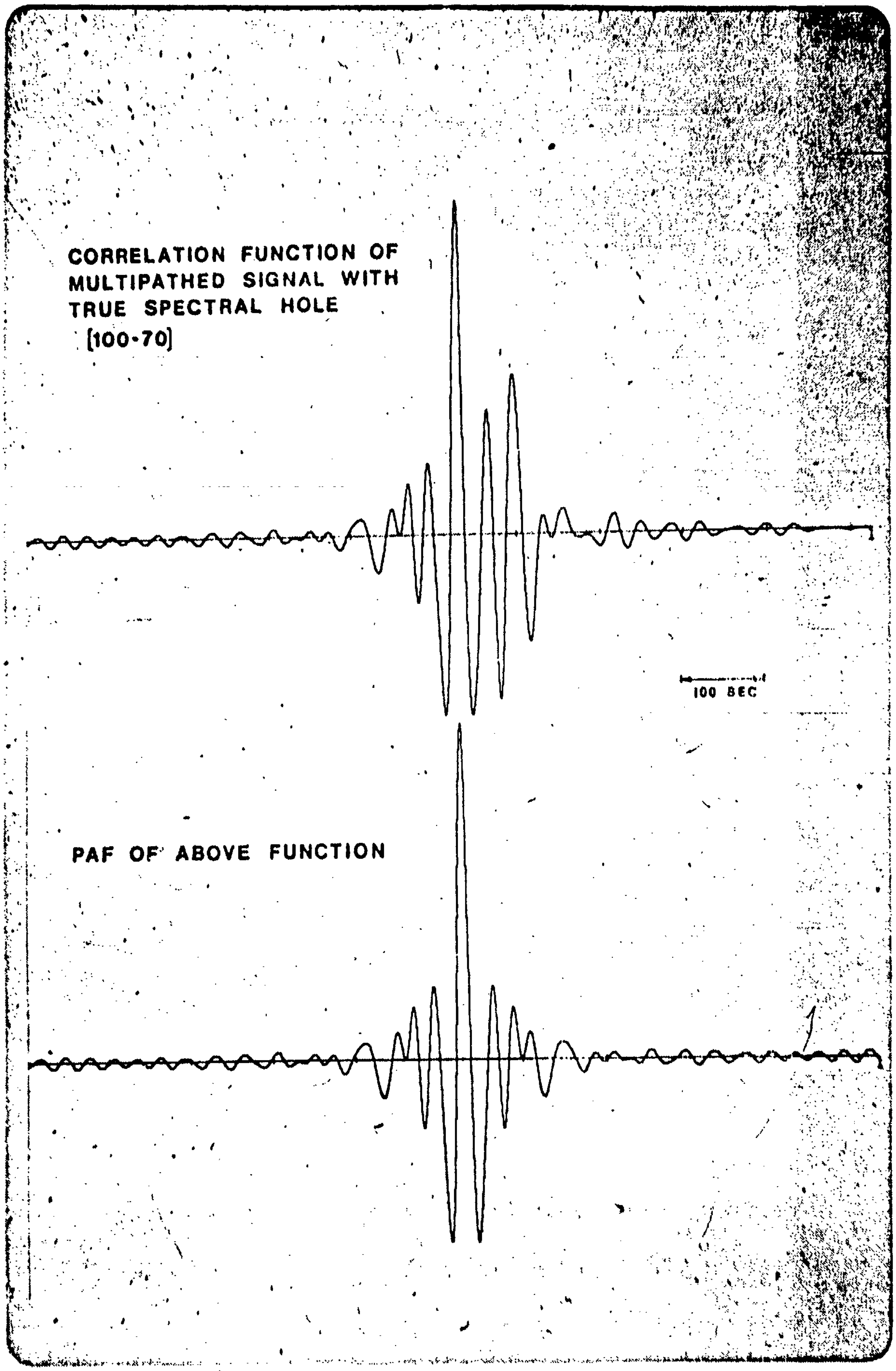

CORRELATION FUNCTION OF
MULTIPATHED SIGNAL WITH
TRUE SPECTRAL HOLE
[100-70]
100 SEC
PAF OF ABOVE FUNCTION

Figure 16

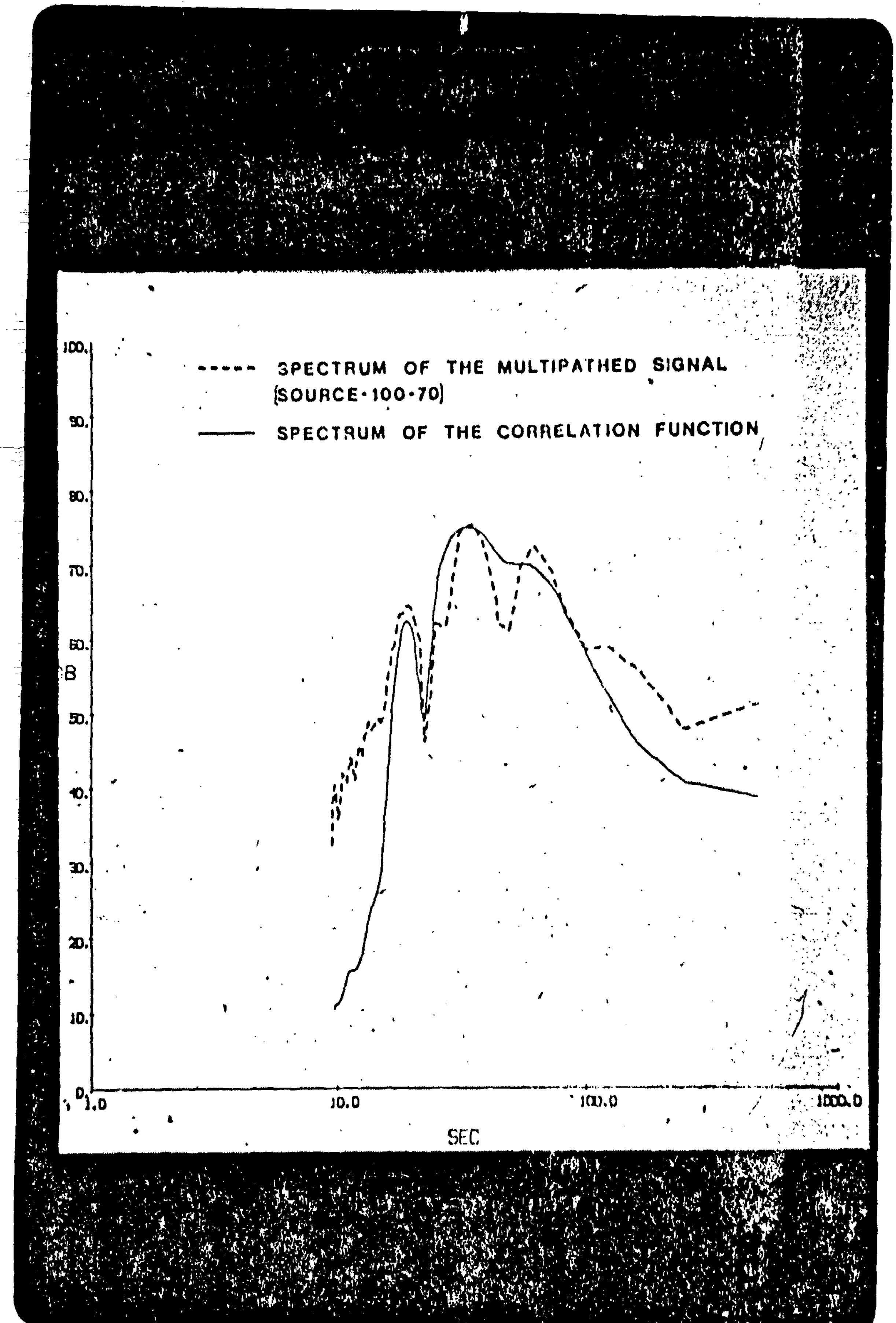

SPECTRUM OF THE MULTIPATHED SIGNAL
(SOURCE·100·70)
SPECTRUM OF THE CORRELATION FUNCTION
100.
90.
80.
70.
60.
DB
50.
40.
30.
20.
10.
0
1.0
10.0
100.0
1000.0
SEC

Figure 17